If G is an n-vertex graph with the chromatic number $\chi(G)$ and the independence number $\alpha(G)$, then

$$\chi(G) \geq \frac{n}{\alpha(G)}.$$

However, this bound might get pretty bad for certain graphs G, and there is in fact no function $g : \mathbb{R} \to \mathbb{R}$ such that $\chi(G) \leq g(v(G)/\alpha(G))$ for every graph G.

The proposed open problem is to find out whether a similar phenomenon happens between the fractional chromatic number $\chi_f(G)$, and the hereditary version of the ratio $v(G)/\alpha(G)$, called the *Hall ratio* and denoted by $\rho(G)$.

Formally,

$$\rho(G) = \max_{H \subseteq G} \frac{v(H)}{\alpha(H)} = \max_{w:V(G)\to\{0,1\}} \frac{\sum_{v\in V} w(v)}{\alpha_w(G)},$$

where $\alpha_w(G)$ is the total weight of a maximum weighted independent set of G with the weight function w. Note that an averaging argument and an LP-duality applied to the LP that computes $\chi_f(G)$ for a given G, one can infer that

$$\chi_f(G) = \max_{w:V(G)\to[0,1]} \frac{\sum_{v\in V} w(v)}{\alpha_w(G)}.$$

In particular, $\chi_f(G) \geq \rho(G)$.

Problem 1. *Is there a function $g : \mathbb{R} \to \mathbb{R}$ such that $\chi_f(G) \leq g(\rho(G))$ for every finite graph G?*

It can be shown that if such a function g exists, then $g(x) = \Omega\left(e^{\log^2(x)/5}\right)$.